



International Alliance for  
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### Statistical Tolerance Intervals and Regions

Statistical intervals and regions, computed based on a random sample, have wide applicability. Confidence intervals and regions, and prediction intervals regions are well-known examples. The topic of the short course is on another type of intervals and regions, namely tolerance intervals and tolerance regions.

A tolerance interval for a univariate population, computed using a random sample, is an interval that will include a certain proportion or more of the population distribution, with a given confidence level. In particular, an upper tolerance limit for a univariate population is such that with a given confidence level, a specified proportion or more of the population distribution will fall below the limit. This proportion is referred to as the content of a tolerance interval. Furthermore, the confidence level associated with the tolerance interval captures the sampling variability. A lower tolerance limit, or a tolerance interval having both lower and upper limits, satisfy similar conditions. For multivariate populations, we analogously have tolerance regions. The theory of statistical tolerance intervals and tolerance regions has undergone vigorous development, starting with the early works of Wilks (1941, 1942) and Wald (1943). A significant amount of recent and very recent literature is also available on the topic, motivated by specific applications and computational considerations. Applications of tolerance intervals and tolerance regions are varied and extensive. They include clinical and industrial applications: quality control, environmental monitoring, the assessment of agreement between two methods or devices, occupational exposure monitoring, the computation of reference intervals and regions in laboratory medicine, and a host of other applications. Starting with the simplest case of a univariate normal distribution, the short course will introduce the participants to the methodological developments and applications of tolerance intervals and regions under various scenarios: regression models, random effects models, multivariate normal models (including multivariate regression models), and non-parametric tolerance intervals and regions. In the multivariate case, the computation of both ellipsoidal and rectangular tolerance regions will be discussed, the latter being motivated by applications in laboratory medicine. Numerous applications will be presented, and computational issues will be briefly addressed.

Some of the material to be presented will be taken from the book *Statistical Tolerance Intervals and Regions: Theory, Applications and Computations* by Krishnamoorthy and Mathew (2009, Wiley). However, a significant part of the short course will include more recent developments on the topic.

